

THE PARADOX OF LIQUIDITY*

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The more liquid a firm's assets, the greater their value in a short-notice liquidation. It is generally thought that a firm should find it easier to raise external finance against more liquid assets. This paper focuses on the dark side of liquidity: greater asset liquidity reduces the firm's ability to commit to a specific course of action. As a result, greater asset liquidity can, in some circumstances, reduce the firm's capacity to raise external finance. Firms with "excessively" liquid assets are in the best position to finance illiquid projects. This leads us to a theory of financial intermediation and disintermediation based on the liquidity of assets.

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I. INTRODUCTION

The liquidity of an asset means the ease with which it can be traded. The more liquid a firm's assets, the greater their value in short-notice sales. Liquid assets are generally viewed as being easier to finance, other things equal. Asset liquidity is, therefore, often a plus for nonfinancial corporations or individual investors. For financial institutions, however, increased liquidity can paradoxically be bad. Although more liquid assets increase the ability to raise cash on short notice, they also reduce management's ability to commit credibly to an investment strategy that protects investors. The problem becomes more acute when the institution is in the business of making markets or trading for its own account.

This paradox of liquidity can be seen in the following example. Consider a firm formed to make markets in government and corporate bonds. It starts with an inventory of liquid Treasury bonds. If these securities could be irrevocably assigned as collateral to a lender, the company could finance its inventory with almost 100 percent debt. But then the inventory would be locked up and of no use in trading.

Such a firm cannot commit not to trade, and the liquidity of its assets opens up various trading strategies, many adverse to the lender. When it is essential for the firm to retain the flexibility to buy and sell assets, the firm may end up too liquid for its financiers' comfort.

A financial institution investing mainly in illiquid business could find long-term financing easier to arrange. Illiquidity means that creditors get less if they seize and sell the assets, but it increases the odds that the assets will "be there," and gives creditors more time to assess their values and risks. In short, liquid assets give creditors greater value in liquidation, but they also give borrowers more freedom to act at creditors' expense. While both issues have been separately recognized, their interactions are largely unexplored.¹

We show that a firm which starts with liquid core assets has an absolute advantage in obtaining external finance for less liquid projects. The incremental external financing the firm generates by taking on the less liquid project exceeds the financing the project can obtain on its own.

¹ For the first issue, see Diamond [1991], Hart and Moore [1994], Myers [1977], and Shleifer and Vishny [1992]. For the second, see Diamond and Dybvig [1986], Esty [1993], Flannery [1994], Huberman [1984], Kahn [1992] and an extensive literature on risk-shifting moral hazards, dating to Jensen and Meckling [1976].

Firms with relatively liquid core businesses are then best suited to channel financing to other firms in the economy. This leads to a theory of financial intermediation which is consistent with the historical origins of the rise of banks -- that banks started by providing payment services and only later moved to lending. Our theory can also explain why disintermediation -- in which the most creditworthy firms bypass banks to borrow directly from the markets -- has increased recently.

Our essential point is quite general. A manager, by virtue of her operational control, has implicit property rights in the firm. Greater asset liquidity enables her to transform assets so as to alter the distribution of implicit property rights in her own favor. At the same time, greater asset liquidity improves a financier's ability to exercise control over the manager and thus enhances his property rights. As a result, greater asset liquidity increases the potential for conflict between the manager and the financier over property rights. This is ultimately resolved by limiting the manager's operational flexibility (and potentially reducing the firm's output), limiting the financier's control rights (and hence reducing financing), or altering the assets that are held together in the firm.

The rest of the paper is as follows. In Section II, we first catalog the various meanings and implications of "liquidity" from the manager and financier's points of view. These depend, for example, on the financier's legal and contractual rights, on the manager's need for flexibility in the sale or use of assets, and on whether the firm is transparent or opaque to outsiders. Then we analyze a simple model illustrating the paradox of liquidity and the determinants of external financing. In Section III, we consider whether the simple model is robust and consistent with optimal contracting. Section IV works out the conditions under which financing is done more efficiently via financial intermediaries. We consider the historical emergence of banks and general implications for the roles of financial markets and intermediaries. We conclude with policy conjectures and suggestions for future research.

II. THE BASIC MODEL

A. Framework

Consider a manager who has a positive net present value investment of \$1 which she wants financed as much as possible by an outside investor. The project requires the purchase and use of a real asset which generates cash flows of C_1 and C_2 at dates 1 and 2.

The asset's depreciated value (its present value in current use or net replacement cost) is d_t at date $t \in \{1,2\}$. We assume risk neutrality and a zero interest rate. Positive net present value requires that $C_1 + C_2 + d_2 > 1$. We also assume that $C_2 + d_2 \geq d_1$, so that it pays to continue the project at date 1.²

There are only two contractible variables. First, the ownership of the asset can be verified and contracted on. Ownership also carries two non-contractible "residual rights:" the right to choose who operates the asset and the right to sell or transfer the asset. Second, cash paid by the manager to the investor (or vice versa) can be verified and contracted on. But the cash flows C_1 and C_2 are *not* verifiable. Once a cash flow is generated, the investor cannot prevent the manager from taking it.

Given what is contractible, the obvious source of external financing is debt. The manager agrees to give up ownership if she defaults on pre-specified payments at dates 1 and 2. (Of course she may attempt to renegotiate the terms of the loan.) The manager might also sell equity, giving unconditional ownership to the outside investor. Equity is feasible if investors can extract part of future cash flow. The equity investor would negotiate at dates 1 and 2, trying to force payout by threatening to fire the manager. But in this simple (no uncertainty) world, equity amounts to debt with promised payments large enough to guarantee default and renegotiation. Therefore we can concentrate on debt financing. This is discussed further in Section III.

The investor cannot operate the asset efficiently. If he gets ownership and chooses to remove the manager from operating control, he will either employ a new manager or sell the asset to a new manager. We focus on the latter action which we term *liquidation*.³ Liquidation gives the investor αd_t , where α , an

²Of course a positive d_2 implies subsequent cash flows. We could keep track of one more cash flow C_3 , for example. But we assume $d_3 = 0$, so that the investor has no bargaining power after date 2. Thus we can stick to a two-period model.

³The former action is, in fact, equivalent to the financier holding a controlling outside equity stake in the firm because the financier retains the right to fire the manager at will. We will discuss the problems with equity financing later.

index of liquidity, is a constant between 0 and 1. The liquidity of cash is $\alpha = 1$.

Liquidity is high if the asset is easy to value and potential buyers have no reason to suspect a "lemon", if it has many potential users who are not wealth-constrained, and if well developed institutions facilitate search and transactions. Our model does not explain liquidity. We just assume that some assets are more liquid than others.

B. Liquidity and Transformation Risk

When the manager operates the assets, she gets the unverifiable cash flows. She may also be able to take or transform the asset. There are several possible scenarios for an investor to worry about.

1. The manager could simply steal the asset and hold or operate it elsewhere. There are many ways to steal, for example, by selling at a bargain price to a relative or affiliate. Anonymous, transportable assets, such as cash, bearer bonds, or commodities, are easier to steal than fixed assets such as land or unique assets such as paintings by old masters.

2. The manager could sell the asset and take the cash, either directly or in the form of perks or excessive salaries. The payoff to the manager is proportional to the asset's liquidity.

3. Some assets turn into cash in the ordinary course of business. Managers can "liquidate" working capital by not plowing the right amount back into inventory. Reinvesting too little in inventories⁴ is difficult to contract against, because optimal inventories depend on potential future sales, which managers can usually forecast better than the investors.

4. Managers could transform liquid general-purpose assets into specific assets that have little value without the manager (Shleifer and Vishny [1989]). By so doing, the manager reduces the investor's ability and incentive to threaten liquidation, and thus transfers value away from the investor. Note that the liquidity of the initial asset facilitates the substitution into illiquid assets.

5. Managers could substitute risky assets for safe ones, shifting value from outside debt investors

⁴ Note the parallel to Myers's [1977] under investment problem. Our point about inventories applies to any depreciating asset that is normally replaced.

to the owner-manager's residual claim (Jensen and Meckling [1976]).

From this point on we will refer to any (or all) of these actions as asset *transformation* and explore how *transformation risk* affects external financing.

The manager's payoff from transforming assets depends on asset liquidity. We assume transformation gives her $\alpha_M d_t$. We assume initially that $\alpha_M = \alpha$, our measure of intrinsic liquidity to an outside investor. It is possible, however, that α_M can be different from α .

In economies with efficient legal systems, certain forms of asset transformation are costly or dangerous for managers. Thus the manager's α will usually be less than the investor's. For example, the expected net payoff from outright theft depends on the costs of trying to conceal it, the probability of capture, and the severity of punishment. The development of institutions other than the legal system can also alter transformation risk. The manager's transformation payoff $\alpha_M d_t$ is reduced if an asset's optimal use can be easily monitored, if public information enables the effective enforcement of contracts, if offsetting incentives can be created -- by stock options, for example -- and if public and private institutions act as incorruptible third-party facilitators.

Even in an advanced institutional setting, transformation risk may still persist, depending on how the asset is used. Consider a very liquid asset, say Treasury bonds, with current U.S. laws and institutions.

1. For a manufacturing company, a Treasury bond creates no transformation risk because it can be "locked up" with a third party custodian (or with the investor) without damaging the company's business.

2. But suppose the manager needs to retain the flexibility to sell or trade assets. Liquid assets may still be excellent collateral if the cost of monitoring is low and the existence and ownership of the assets can be verified at frequent intervals. Money market mutual funds holding Treasuries are an example. The funds are virtually risk-free, although they trade frequently. Disclosure rules, and regulatory restriction of investment to a narrow asset class makes these funds nearly *transparent*; that is, portfolio composition is public knowledge (at least once a quarter) and deviations from pre-established investment policies are

relatively easy to spot.⁵

3. But when the manager's core business is trading, not just into Treasuries, but other assets and possibly derivatives, deviations from value-maximizing strategies are hard to detect. The manager's portfolio may fluctuate minute by minute. Moreover, while individual assets may be easy to understand, the portfolio may not be: it is not easy for an outsider to step in and "unwind" a complicated position. Unless the outside investor can track the position, and has the expertise to evaluate it, monitoring is useless. So effective monitoring is costly. More important, it would reveal the manager's expectations and tempt the expert investor to front-run her. All these factors combine to make the portfolio nearly opaque to outside investors.

Security traders and dealers therefore face the paradox of liquidity in its most extreme form. They hold highly liquid assets in rapidly changing mixtures and take on complex positions. They cannot reveal short-run strategies. This combination of liquidity and opaqueness can put the highest credit ratings out of reach for long-term finance.⁶

Therefore, while improvements in government regulation and the development of laws and institutions raise the cost of some forms of asset transformation, they have little effect on other forms. Moreover, improvements in markets have made many assets more liquid, and thus increased the opportunities to transform assets. The net effect of financial and economic development on managers' incentives for asset transformation does not always favor outside investors.

We believe the paradox of liquidity arises in many contexts in all stages of economic and financial development. The only systematic exceptions are in primitive economies (or perhaps criminal industries) where basic property rights are not enforced and nothing is contractible. Here liquidity is irrelevant regardless of how assets are used, because outside investors have no check on what managers can do. In these extreme cases, all assets, even the most illiquid ones, are expropriable.

⁵ The importance of transparency (versus opaqueness) to outside investors is stressed by Merton [1995].

⁶Short-term finance may still work, for example through repurchase agreements. The trader may be able to use securities as temporary collateral, retaining flexibility to recover and sell the securities on short notice.

Our modeling strategy is as follows. We keep the institutional structure in the background, implicitly holding it constant, and vary asset liquidity by changing α . We solve for the maximum amount of external financing the manager can raise, assuming that she may transform assets as well as appropriate cash flow. We analyze the relationship between asset liquidity and the ability to raise external finance.

In section III we recognize that the institutional structure is partly endogenous. In other words, the manager may be able to design covenants, collateral arrangements or monitoring to reduce or eliminate transformation risk. We model this by allowing the manager to choose α_M ex ante. We also explicitly model transformation through asset substitution rather than misappropriation to show our results generalize.

C. Financing and Liquidation

The manager raises B from the investor and makes up any shortfall with E dollars of her own funds. The financing contract specifies B and the repayments P_1 and P_2 due at dates 1 and 2. It also transfers ownership to the investor if the payment is not made and the contract is not renegotiated (see below).

Both manager and investor contemplate liquidation. The investor may have to sell the assets at dates 1 or 2 if the manager does not make the (potentially renegotiated) required repayment. The manager may choose transform the assets before the investor can get them, that is, at dates $1/2$ or $1\ 1/2$. We assume that assets cannot be partially transformed. Also, the investor is not around at the intermediate dates to thwart transformation.

We allow for contract renegotiation at date 1 and 2, following Hart and Moore's [1994] framework: at each date, the manager either makes the specified payment or offers to make a lower payment. If the investor turns down the offer, he can either liquidate the firm or continue the renegotiation process. If he continues negotiating, he gives up the right to liquidate the firm for the rest of the period.

If the investor negotiates, he has a probability a -- a measure of his bargaining power -- of making the next (and final) offer. With probability $(1-a)$ the manager gets to make this offer. If the final offer is turned down, the next period's cash flow (for example, cash flow C_2 at date 1) is not produced. If negotiations break down, the unpaid promised payment cumulates to the next date when negotiations resume.

The timing of manager's and investor's decisions is:

Date 0	Date 1/2	Date 1	Date 1 1/2	Date 2
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Investment undertaken after raising B from investor.	Management decides whether to transform assets and realize αd_1 .	C_1 realized. Investor can liquidate assets if management does not make contractual payment and manager has not transformed assets.	Management decides whether to transform assets and realize αd_2 .	C_2 realized. Investor can liquidate assets if management does not make contractual payment and manager has not transformed assets. If manager retains possession of assets, she realizes d_2 .

Except for the threat of asset transformation at dates 1/2 and 1 1/2, our setup and assumptions match Hart and Moore [1994].

We will determine the maximum amount that can be financed. The project will never be inefficiently liquidated at this "financing capacity" provided the manager has enough cash to make the payments that can be credibly extracted by the investor. We want to focus on the tension asset liquidity creates between the manager's willingness to pay and the investor's ability to extract repayment. We will not focus on the complications introduced by the borrower's inability to pay the amount that can be extracted if the timing of project cash flows is skewed towards the future (for this, see Hart and Moore [1994]). Therefore, we assume the manager has access to cash flow from other (un-modeled) sources with which to make payments if current and past cash flow from the project are insufficient. Of course, the sum of cash inflows from the project will be sufficient to make the required payments. Given this framework, the manager's capacity to obtain external finance is not monotonic in the intrinsic liquidity of the assets. Liquidity can be a double-edged sword.

D. Capacity for External Finance

We first determine the outcome of any renegotiation at date 2. If the manager makes the payment P_2 (which may have been renegotiated at date 1), she keeps the assets and gets d_2 . If she does not make the

payment, the investor can liquidate the assets. So the outcome of the bargaining at date 2 is simply that the debt claim P_2 is renegotiated down to $\text{Min}[P_2, \alpha d_2]$.

Now move back to date 1 1/2, when the manager has to decide whether to transform the assets, realizing αd_2 , or continue in business, generating cash flow C_2 and the terminal value d_2 . She continues if

$$(1) \quad C_2 + d_2 - \text{Min}[P_2, \alpha d_2] \geq \alpha d_2$$

This implies that the maximum date-2 repayment the investor can get, with the assurance the manager will not transform the assets, is

$$(2) \quad V_2^L = \text{Min}[C_2 + d_2 - \alpha d_2, \alpha d_2]$$

Now consider what happens when the promised payment is renegotiated at date 1. This is a simple restatement of lemma 1 in Hart and Moore [1994].

LEMMA 1. If the manager asks to reschedule payments at date 1, the investor can get at maximum

$V_1^L = \text{Max}[aC_2 + V_2^L, \alpha d_1]$. Since the manager has enough cash to make extractable payments, she will get $C_1 + C_2 + d_2 - V_1^L$ from the project where

$$(3) \quad P_1^* + P_2^* \leq V_1^L$$

$$(4) \quad P_2^* \leq V_2^L$$

The intuition is simple. Consider the negotiation that takes place at date 1 if the manager does not make the required repayment. Suppose the manager makes an offer which the investor turns down. With probability a , the investor gets to make a take it or leave it offer. The investor will demand C_2 , the entire cash flow that is to be produced over the period. The manager can do no better than accept, because if the offer is rejected, the assets remain idle over the period. With probability $(1-a)$, the manager gets to make a take it or leave it offer, and will demand the entire cash flow from the project C_2 . Again, the investor can do no better than accept. On average, the investor and manager split the cash flow according to their relative bargaining power. So the total stream of payments the investor can credibly get (the continuation value) is $aC_2 + V_2^L$. In addition, the investor can force liquidation immediately, so $V_1^L = \text{Max}[aC_2 + V_2^L, \alpha d_1]$, with

liquidation placing a lower bound on what the investor can get. If the manager has the cash, this is what she will offer to pay, and the offer will be accepted.⁷ Inequality (3) and (4) simply indicate a credible payment schedule.

We now know what happens at date 1 if the manager attempts to renegotiate the initial contract. But the manager can consume or transform the assets at date 1/2. This puts a further constraint on the amount she can promise. Let $V_{1/2}^L$ be the amount that the manager expects to pay the investor over the course of the contract (after allowing for the equilibrium outcomes of any possible renegotiation at dates 1 and 2). The manager can commit not to transform the assets only if:

$$(5) \quad C_1 + C_2 + d_2 - V_{1/2}^L > \alpha d_1.$$

Finally, the investor will offer financing at date 0, only if:

$$(6) \quad V_{1/2}^L \geq B$$

The maximum debt that the manager can credibly commit to repay is then the solution to the following linear programming problem. Each constraint is keyed to a renegotiation or possible transformation.

Max $B = P_1^* + P_2^*$, subject to:

$$(7) \quad P_1^* + P_2^* \leq C_1 + C_2 + d_2 - \alpha d_1 \quad \text{date 1/2 transformation}$$

$$(8) \quad P_1^* + P_2^* \leq \text{Max} [aC_2 + \text{Min}[\alpha d_2, C_2 + d_2 - \alpha d_2], \alpha d_1] \quad \text{date 1 renegotiation}$$

$$(9) \quad P_2^* \leq C_2 + d_2 - \alpha d_2 \quad \text{date 1 1/2 transformation}$$

$$(10) \quad P_2^* \leq \alpha d_2 \quad \text{date 2 renegotiation}$$

Transformation risk at date 1/2. We concentrate first on constraints (7) and (8). Constraint (9) will not bind if C_2 is large enough, and if enough cash is available at date 1 to front-load payments in P_1^* , rather than P_2^* . In this case constraint (8) reduces to

⁷ If the terminal value of the asset, d_2 , comes from a future (unmodelled) cash flow C_3 from the asset, then the investor cannot get more than αd_2 if $aC_3 < \alpha d_2$. This is implicit in our solution to the renegotiation outcome at date 2.

$$(8a) \quad P_1^* + P_2^* \leq \text{Max} [aC_2 + \alpha d_2, \alpha d_1]$$

If only constraint (8a) were relevant, financing capacity would be (weakly) monotonically increasing in the intrinsic liquidity of assets α . (See Figure I.) Monotonicity no longer holds when the transformation risk constraint (7) is added. Financing capacity decreases in asset liquidity when this constraint is binding. Constraint (7) binds if $C_1 + C_2 + d_2 - \alpha d_1 < \text{Max} [aC_2 + \alpha d_2, \alpha d_1]$ for $\alpha < 1$. This occurs when the cash flows C_1 and C_2 are not too high relative to the liquidation value.

X-----Figure I approximately here-----X

E. Financing Capacity Contributed by a Project

The contract studied above, with finite pre-specified payments and change in ownership contingent on payments not being made, amounts to external debt. Previous literature has focused on situations where the manager does not transform or consume the assets, whence the assertion that debt capacity increases in asset liquidity. We now examine the effects of liquidity on debt capacity when a firm with a specific asset structure wants to undertake a new project. We show that debt capacities do not necessarily add up. Debt capacity can be substantially greater (or less) when two projects are undertaken together rather than separately.⁸

Example: Consider a project with $C_1^P = 1, C_2^P = 1, d_1^P = 2, d_2^P = 1, a^P = .5, \alpha^P = .3$. Assume the date-0 investment required is 2. The project can support debt up to its "going concern" debt capacity, $a^P C_2^P + \alpha^P d_2^P = .8$. Because the cash flows are large relative to liquidation value, transformation risk does not bind. But because liquidation value is low relative to the cash flows, the cash flows do not contribute much to debt capacity since the investor cannot enforce payment. The project has insufficient collateral -- it is "illiquid".

⁸ This is different from the well-known result that diversification reduces bankruptcy costs and increases debt capacity. That result is uni-directional (more diversification always increases debt capacity). Our result can go either way. Furthermore, we obtain this result in a world of certainty. With uncertainty, it is easy to show that diversification helps the manager commit to not transform the assets. Kahn [1992] also describes interactions between projects. We differ in our emphasis on liquidity.

Suppose a firm invests 5, yielding $C_1^F = 1$, $C_2^F = 1$, $d_1^F = 5$, $d_2^F = 4$, $a^F = .5$, $\alpha^F = .6$. The intrinsic liquidity of the firm's assets is higher than the project's, though its cash flows are relatively small compared to the asset value. Its debt capacity, determined by the liquidation value of its assets, is $\alpha^F d_1^F = 3$.

Now consider what happens when the firm undertakes the project in-house. Debt capacity for the combined entity turns out to be $a^P C_2^P + a^F C_2^F + \alpha^P d_2^P + \alpha^F d_2^F = 3.7$. This is less than the sum of the debt capacities of the stand-alone firm and project (.8+3=3.8), since the maximum of the sum is (weakly) less than the sum of maximums. Because the combined debt capacity is not determined by liquidation value, the firm will not be able to contribute its entire stand-alone debt capacity, which is largely determined by liquidation value.

But let the firm's assets be more liquid, so that $\alpha^F = 1$. Transformation risk now limits the firm's stand-alone debt capacity to $C_1^F + C_2^F + d_2^F - \alpha^F d_1^F = 1$, even though the liquidation value of the assets is higher at $\alpha^F d_1^F = 5$. This implies that the firm has liquid assets which cannot be pledged because the firm cannot commit to not transforming assets (after date 1/2) if it borrows too much. The firm is "overly liquid".

When the overly liquid firm undertakes the project, combined debt capacity is determined by the lower of liquidation value and transformation risk, which is 3.4. This is much higher than the sum of stand-alone debt capacities (.8+1=1.8). The project has cash flows, but not enough liquidation value for the investors to extract repayment. The firm has excess liquidation value but too little cash flow to commit not to transform the assets. When the firm undertakes the project, the project's cash flow insures the investor against transformation risk (alleviates the transformation risk constraint), while the firm's liquidation value enables the investor to credibly extract repayment. Combining the firm and project increases debt capacity.

An important assumption in this example is that assets cannot be liquidated or transformed separately. We will discuss this shortly. But first we formalize the example. Each constraint in the linear program can bind over a range of intrinsic liquidity α . The three ranges are

- (a) $0 \leq \alpha \leq \alpha_L$, where $B = aC_2 + \alpha d_2$
- (b) $\alpha_L < \alpha \leq \alpha_{LL}$, where $B = \alpha d_1$

(c) $\alpha_{LL} < \alpha \leq 1$, where $B = C_1 + C_2 + d_2 - \alpha d_1$

If condition (a) holds, we call the firm or project *illiquid* because debt capacity depends cash flows. If condition (b) holds, it is *liquid* because debt capacity is fully determined by the liquidation value of the assets. If condition (c) holds, it is *overly liquid* because its debt capacity is reduced by its excessive liquidity. The relevant ranges are shown in Figure II.

X-----Figure II approximately here -----X

When a firm takes on a project, if the assets cannot be liquidated or transformed separately, we have the following result.

PROPOSITION 1. a) The incremental debt capacity an illiquid, liquid or overly liquid firm gets when it takes on an illiquid, liquid or overly liquid project, respectively, is equal to the project's debt capacity when financed independently.

b) The incremental debt capacity a liquid firm gets when it takes on an illiquid project is (weakly) less than the project's debt capacity when financed independently.

c) There always exists a range of illiquid and liquid projects such that the incremental debt capacity the overly liquid firm obtains with those projects strictly exceeds the project's debt capacity when financed independently.

PROOF OF PROPOSITION 1. See appendix.

Thus it makes more sense for the *overly liquid* firm to undertake less liquid projects than for such projects to raise finance directly from investors. The overly liquid firm has an excess of liquid assets which cannot be collateralized because they are too liquid, that is, too easily transformed. The illiquid project has an excess of cash flows and a deficit of liquid asset value. The excess liquid assets of the overly liquid firm serve to collateralize the illiquid project while the illiquid project's cash flows serve to bind assets in place and reduce transformation risk. By contrast, the illiquid project brings insufficient assets to a *liquid* firm, while the liquid firm brings insufficient cash flows to the project, so they are worse off borrowing together than apart.

As the liquidity of the assets of an overly liquid firm increases, its ability to finance projects of lower liquidity generally increases. Let $[\underline{\alpha}^P, \overline{\alpha}^P]$ be the range of intrinsic liquidity of projects for which there is a greater increment in debt capacity in doing the project in-house relative to doing it independently. Furthermore, let α^{*P} be the intrinsic project liquidity for which the advantage is largest (if this is a range, let it be the maximum of the range). Proposition 1 implies:

COROLLARY 1. An increase in the intrinsic liquidity of the overly liquid firm's assets leads to a (weak) decline in $\underline{\alpha}^P$, no change in $\overline{\alpha}^P$ and a (weak) decline in α^{*P} .

PROOF OF COROLLARY 1. See Appendix.

Note the importance of the assumption that the manager cannot transform only some assets or that the investor cannot liquidate them separately. If this were possible, the project and the firm would not insure each other (or have adverse spillover effects on each other). Although this assumption is relaxed in Sections III and IV, it may nevertheless be plausible. For instance, the assets may complement each other in the production of cash flows, so that when one is transformed or liquidated, the cash flow or liquidation value of the other falls dramatically.

III. ROBUSTNESS

A. The Financing Contract

So far we have considered only debt financing, that is, a financing contract giving the investor ownership only if the manager does not make a pre-specified payment. We now argue that debt is the optimal contract.

The investor needs some ownership rights in order to extract repayment. Debt gives ownership contingent on default. The only alternative is equity, that is, a financing contract where the investor has unconditional ownership from the beginning. In our setup, where uncertainty is excluded⁹ and only pre-

⁹Debt and equity financing are more clearly distinguished in models with uncertainty and infinite horizons. See Fluck [1995] and Myers [1996]. As in Hart and Moore [1994], we do not allow contracts contingent on date 1 events (as opposed to contracts contingent on payments). Thus we rule out Golden Parachutes which would pay the manager a large sum if she were fired.

specified payments and ownership are contractible, equity is equivalent to a debt contract with promised payments large enough to guarantee default and bargaining at dates 1 and 2.

For illiquid or liquid assets, where financing capacity is determined by the anticipated outcomes of bargaining at dates 1 and 2, debt and equity are essentially equivalent.¹⁰ For excessively liquid assets, where financing capacity is determined by the transformation constraint (7), debt dominates equity. The reason is interesting. If (7) binds, the debt payments P^*_1 and P^*_2 are set below the amounts that the investor could extract, on average, from bargaining. This is necessary to ensure the manager will not transform. Since equity payments are set only through bargaining, no such ex ante limitations can be placed on them, and the manager will transform the asset at date 1/2. Thus, if the investor has a controlling equity interest, and can be expected to bargain for future payment, transformation risk is increased and the equity is worthless.¹¹ Debt has value, however, because it limits the power of investor only to states of default. Thus protected, the manager does not transform assets.

More generally, the manager in operational control has the ability to transform assets. This “outside” option necessitates promising her some of the future cash flows. The investor also has contingent ownership rights which gives him the “outside” option of liquidating assets. Increasing asset liquidity enhances the value of both outside options, so much so that they can exceed the value of co-operating. This is the situation in an overly liquid firm where the manager may want to exercise her outside option before the investor gets a chance to exercise his. Since the manager’s ability to transform cannot be checked contractually (thus far), and total cash flows are fixed, promised payments have to be

¹⁰The payout to equity is not determined until bargaining is complete, but anticipated payments, which determine financing capacity, are still given by Eqs. (8) and (10). If there are any costs of bargaining, debt is better than equity.

¹¹ This may seem counter-intuitive to those who regard equity as “softer” than debt. But equity has unconditional control, and therefore can be “harder” than debt, which has limited and contingent rights. This is another reason why the investor would sell the asset to a new owner-manager rather than keeping the asset and employing a manager to run it. The manager’s incentive to transform assets is higher when the investor enjoys the unconditional powers of ownership.

pre-specified and set low enough to eliminate transformation risk.¹²

The manager's incentive to transform is reduced when illiquid projects are added. Ownership rights give the investor less power when projects are illiquid. Adding illiquid projects also reduces the manager's ability to transform. So there is surplus left over even after accounting for each agent's outside option (the manager's option to transform, and the investor's option to liquidate). Combining an overly liquid project with a illiquid project can thus enhance the manager's ability to finance externally. **B. The Manager's Flexibility**

Thus far, we have assumed no difference between the payoffs to transformation by the manager and liquidation by the investor. This is implausible because the manager may voluntarily give up the flexibility to transform assets in order to get more external finance. Suppose the manager's $\alpha_M = b\alpha$ where $b \in [0,1]$ is the manager's *flexibility* which she can reduce by writing contracts pledging assets to investors, or third parties, or by allowing interim monitoring and public audits.

There are constraints, however, on the extent to which the manager can reduce b . Some assets are virtually impossible to monitor or pledge. For example, cash's b and α will often be close to one: cash can readily be absorbed in perks and bonuses. Public institutions and authorities may not help attempts to reduce transformation risk if they are corrupt or unsophisticated.

More important, even when low b 's are feasible they are not always desirable, because the cash flows generated by the assets will also depend on the manager's flexibility. This is especially true if cash flows are generated by transforming assets, as in securities trading. We model this as follows. Let cash flows C_1 and C_2 be reduced by the fraction $f(b, \alpha)$, with $f_b > 0$ (subscripts indicate partials) and $f(1, \alpha) = 1$. So more flexibility increases cash flow, and cash flow is maximized when $b = 1$.

In choosing optimal flexibility, the manager trades off the effect of flexibility on cash flows against its effect on external financing. Of course, lower external financing is costly only if the alternative

¹²This argument does not depend on relative bargaining power, a . Contractual schemes to alter bargaining power do not mitigate the fundamental problem.

source of finance, the manager's personal funds, have a high opportunity cost. Since every enterprise the manager undertakes requires her to invest some of her limited wealth the opportunity cost is the surplus from other enterprises foregone. Let the additional cost of a dollar of personal investment (compared to a dollar of external debt) be θ . We assume here that the transformation constraint binds at date 1 for all relevant values of b . Then the manager maximizes

$$(11) \quad f(b, \alpha)(C_1 + C_2) - \theta [1 - (f(b, \alpha)(C_1 + C_2) + d_2 - b\alpha d_1)]$$

with respect to b . The first term in (11) is the (reduced) cash flow generated by the project. The second term (in square brackets) is the financing shortfall which has to be covered from personal wealth.

Differentiating with respect to b , we get

$$(12) \quad (1 + \theta) f_b(b, \alpha)(C_1 + C_2) - \theta d_1$$

If f_b is high enough, so that flexibility is very important for the generation of cash flows -- as for a securities trader -- the manager will set $b = 1$. If f_b is small so that cash flows do not depend much on flexibility -- as in the case of machinery built into an assembly line -- the manager will set b as low as possible. Assume f is concave. Then if there is an interior optimum flexibility, b^* , it increases with cash flows, decreases with θ , the excess opportunity cost of personal wealth, and decreases with the date-1 value of assets, d_1 .

The effect of a change in intrinsic liquidity on b^* is more subtle. Implicitly differentiating after setting (12) equal to zero, we get

$$(13) \quad \frac{db^*}{d\alpha} = - \frac{[(1 + \theta) f_{b\alpha}(b^*, \alpha)(C_1 + C_2) - \theta d_1]}{(1 + \theta) f_{bb}(b^*, \alpha)(C_1 + C_2)}$$

Because f is concave, the relationship between optimal flexibility and liquidity depends on the sign of the expression in square brackets. For businesses that "buy and hold" assets, liquidity will not affect cash flows, and $f_{b\alpha}$ will be small. In this case, an increase in asset liquidity primarily increases the marginal

impact of flexibility on debt capacity. By agreeing to lower flexibility, the manager can increase debt capacity more when asset liquidity is high. Thus $db^*/d\alpha$ is negative. But if a firm is in the business of trading assets, periods of high liquidity are precisely when the assets are least costly to trade, and the most money can be made. The impact of flexibility on cash flow is highest when liquidity is high. For such businesses, it is possible that $f_{b\alpha} \gg 0$, and increased asset liquidity can imply higher optimal flexibility.

C. Transformation Risk at date 1 1/2 and the Maturity of Debt Capacity

Thus far, we have assumed that transformation risk does not bind at date 1 1/2. Suppose now that re-payments cannot be advanced to date 1, perhaps because of cash flow constraints. Second-period debt capacity is given by $\text{Min} [C_2 + d_2 - b\alpha d_2, \alpha d_2]$, where the first term is the transformation risk constraint at date 1 1/2 and the second is the liquidation value of the assets.

We can now say something about the maturity of debt capacity. Hart and Moore [1994] show that slower asset depreciation means longer maturity debt. When assets retain value, the investor retains the ability to extract repayment, and can lend longer-term. Assets such as land, which do not depreciate ($d_1 = d_2 = 1$), can support the longest-term debt.

Land is easily pledged. It is impossible to make away with land and sell it without title, or to transform it. So flexibility can be made very low, and liquidation value will determine debt capacity at date 2. Other things equal, an increase in the intrinsic liquidity of land will (weakly) increase the ratio of second-period debt capacity to total debt capacity, i.e., increase the maturity of debt capacity.¹³

But other assets, such as inventories, which do not depreciate because they are continuously replaced, do not support long term debt. Inventories are easily converted into cash simply by not re-investing in them. It is hard to guard against this without unduly limiting the manager's flexibility or incurring high monitoring costs. Optimal flexibility is high for these assets. Also, the transformation risk

¹³ If the firm is illiquid in the first period, this ratio is $\alpha/(aC_2 + \alpha)$ which increases with asset liquidity, α . If the firm is liquid in the first period, the ratio is $\alpha/\alpha = 1$, which does not change with asset liquidity.

constraint is tighter in the second period when these assets continue to retain value and liquidity, but early cash flows have been harvested.¹⁴ So while liquid, non-depreciating assets with low b can support a constant level of debt each period, liquid non-depreciating assets with high b support a decreasing amount of debt in later periods, and have a lower maturity for debt capacity. The maturity shortens even further with increases in intrinsic asset liquidity.¹⁵

D. Asset Substitution and Transformation Risk

Taken literally, our model says that the manager steals the proceeds if assets are liquidated. This is shorthand for the many ways that a manager can use her operating control to reduce the cash that investors can extract. It is interesting to model one of these ways in more detail; The manager could substitute general purpose assets that have high value in the hands of an alternative user into specific assets that have little value without the manager — what Shleifer and Vishny [1989] call entrenching investment. The illiquid specific assets reduce the value of the investor's control rights and, hence, the repayment he can extract. We now show that the manager's incentive to substitute into specific assets increases with the liquidity of the original asset, and we get a transformation constraint similar to (7). Thus, our entire analysis can be couched in terms of asset specificity and substitution. This type of transformation risk is not illegal and confronts managers so long as they have to transform assets in the normal course of business.

Suppose the manager can liquidate an asset for cash at date $1/2$ and re-deploy the proceeds in a new asset that pays cash flows of $\gamma C_t/d_1$ at time t for every dollar invested, where C_t is the cash flow produced by the original project and $\gamma \in (0,1]$ is a parameter. The new asset is specific to the manager

¹⁴ The transformation constraint at date $1/2$ is $C_1 + C_2 + d_2 - b\alpha$. At date $1 1/2$, it is only $C_2 + d_2 - b\alpha$.

¹⁵ If transformation risk binds at date $1 1/2$, second period debt capacity is given by $C_2 + d_2 - b\alpha$, and the maturity of debt capacity decreases with increases in asset liquidity so long as the firm is illiquid or liquid in the first period. But this is true even if the transformation risk constraint (8) binds in the first period. The ratio of second period debt capacity to first period debt capacity is then $(C_2 + d_2 - b\alpha)/(C_1 + C_2 + d_2 - b\alpha)$ which decreases in α .

and has no value d_1 in alternative use.¹⁶

Now consider the manager's decision at date 1/2. She can sell the original asset for αd_1 and invest the proceeds in the substitute project, generating cash flows of $\alpha \gamma [C_1 + C_2 + d_2]$ --recall that d_2 is the unmodelled cash flow at date 3. If (8a) was binding prior to substitution, she only has to pay $a\alpha \gamma C_2$ now, since the substitute project has no liquidation value. Thus the manager trades off the lower cash flows from the substitute project ($\alpha \gamma < 1$) against the reduction in payments the creditor can extract because the project has no liquidation value ($a\alpha \gamma C_2 < \text{Max}[aC_2 + \alpha d_2, \alpha d_1]$). We have a substitution constraint

$$(14) \quad C_1 + C_2 + d_2 - \alpha \gamma (C_1 + C_2 + d_2) \geq (P_1^* + P_2^*) - a (C_2$$

where the left hand side is the reduction in cash flows (the cost of substitution) while the right hand side is the reduction in expected repayment (the benefit of substitution). Rearranging gives

$$(15) \quad P_1^* + P_2^* \leq C_1 + C_2 + d_2 - \alpha \gamma (C_1 + (1 - \alpha \gamma) C_2 + d_2]$$

which has the same form as the transformation constraint (7), with substitution risk increasing with the liquidity α . Thus debt capacity is again non monotonic in asset liquidity.

Now consider a firm that contemplates undertaking a project for which the transformation risk constraint, (15), constrains debt capacity. Assume the firm is large relative to the project. If the firm's debt capacity is determined by liquidation value prior to substitution (*i.e.*, $\alpha^F d_1^F > aC_2^F + \alpha^F d_2^F$), then the cash flows from the project after substitution do not add to the payment that can be extracted by the lender (since the substitution of the small project's assets do not change the fact that combined debt capacity is determined by liquidation value). Therefore, the debt capacity contributed by the project is

$$(16) \quad P_1^* + P_2^* \leq C_1 + C_2 + d_2 - \alpha \gamma (C_1 + C_2 + d_2]$$

¹⁶The assumptions of zero liquidation value and a substitution opportunity only at date 1/2 are for simplicity. More complicated situations can be handled at the expense of additional notation.

By contrast, if the firm is illiquid, the project's cash flows after substitution do add to the payment that can be extracted by the lender (which is $aC_2^F + \alpha^F d_2^F + a\alpha \gamma C_1$ after substitution) so that the debt capacity the project contributes to the illiquid firm(given by (15)) is higher than what it contributes to the liquid firm. The terms "project" and "firm" can clearly be interchanged and we get, substantially, the content of proposition 1.¹⁷

IV. FINANCIAL INTERMEDIATION

To review the basic insights developed so far; Transformation risk means that debt capacity is not monotonic in the intrinsic liquidity of assets. Because the assets of an overly liquid firm cannot be fully utilized as collateral, debt capacity is increased by taking on less liquid projects that, ex ante, are cash flow-rich but asset-poor.

We now apply our model to banking. Why is it that banks have historically held a mix of illiquid loans and liquid securities as assets and short-term deposits as liabilities? If the objective is to meet the cash demands of investors, why not hold only liquid assets? If the objective is to finance long term investment, why not finance through long-term liabilities? Furthermore, why is an intermediary necessary between the ultimate borrower and the initial saver? Consider our explanation. Suppose individuals want to store money safely, and also would like the ability to withdraw cash or make payments. A bank can realize scale economies from offering individuals the ability to hold demand deposits.¹⁸ But it has to hold liquid assets to meet depositors' demands for cash. If the bank holds only liquid assets, transformation risk is likely to limit its debt capacity. So the bank has a comparative

¹⁷ We say substantially because the illiquid firm cannot get more debt capacity for the liquid project than the latter can on a stand alone basis unlike part c of Proposition 1. It is easy, however, to add plausible additional structure to the model such as making the creditor's a dependent on the volume of negotiation he does with the borrower (see next section) to retrieve the implications of part c. Also, though the project can be transformed separately from the firm, the creditor is still assumed to threaten simultaneous liquidation. This makes sense if liquidation means replacing the common manager, or placing the combined entity in bankruptcy.

¹⁸ The aggregate demand for cash by a large number of individuals is more predictable than the individual demands. Also, if the bank maintains more accounts, it can settle more payments internally by book entry, reducing transactions costs. There may also be economies in setting up institutional arrangements to safeguard cash, access credit, or, as Goodfriend [1991] argues, to make final settlements to other banks.

advantage in undertaking and financing less liquid projects.

This suggests that a bank offering payment services would have a natural advantage in taking on an illiquid project in-house. It does not say that the bank would have any advantage in *lending* to the project. Lending by an intermediary introduces a new layer of agency costs between the manager of the project (the ultimate borrower) and the bank providing deposit services. It turns out, however, that if the bank has greater bargaining power than individuals vs. the manager of the project, intermediation can increase the overall amount of finance projects can get. This is what we now show.

A. Intermediation

We model the intermediary as a bank that borrows from a number of individual investors for its own core business and to lend on to a project. A single large outside investor in a project clearly has some advantages over a group of individual investors, each making a small loan. The large investor brings scale economies to monitoring and does not suffer from the problems of organizing collective action that would plague the group of small investors. We therefore assume that the bank has greater bargaining power over project cash flows than individuals, so that $a_F^P > a_I^P$.¹⁹ The subscript denotes lender type, where I represents "individual investor" and F the financial intermediary, the bank.

Even though the bank can extract more from the ultimate borrower, the bank has to finance these loans by borrowing from individual investors. Unless the individuals can force the bank to pay out part of the additional cash it extracts from the project, intermediation does not increase the flow of financing from the individual saver to the ultimate borrower.²⁰ We will show that when intermediation is conducted

¹⁹ Kindleberger [1993, p. 47] suggests that historically, "the small lender needed the security of the intermediary's bargaining power since the latter was much better placed as a rule to collect repayment in the event of difficulty."

²⁰ For the most part, theories of financial intermediation differ only on how this happens. For instance, in Diamond [1984], the intermediary lends to, and monitors, projects on behalf of multiple individuals, thus eliminating their need to individually incur the costs of monitoring projects. This is similar to our assumption that $a_F^P > a_I^P$. However, it raises the question of "who monitors the monitor". In Diamond's model, since the intermediary is diversified across a large number of loans, individuals need only observe the macroeconomic risks the intermediary is subject to -- which are public knowledge -- to fully determine the intermediary's profits. As a result, individuals have no need to monitor diversified intermediaries, and Diamond ingeniously finesses the problem of who monitors the monitor. Of course, if the intermediary could privately choose its exposure to

by an overly liquid bank, the ability of individual investors to threaten to liquidate the bank enables them to extract a portion of the enhanced receipts from the ultimate borrower, despite their own lack of bargaining power. Thus intermediation can increase external funding of projects, and will be undertaken by intermediaries whose core business is overly liquid.

We will focus on an illiquid project that needs financing, but the results are readily extended to other kinds of projects. The debt capacity of the project when funded directly by individual investors is $a_1^P C_2 + \alpha^P d_2^P$. The debt capacity of the project when the bank lends to it is $a_F^P C_2 + \alpha^P d_2^P$. Let this be the amount the bank lends and let the contracted, enforceable repayments be P_1^P and P_2^P . Because $a_F^P > a_1^P$ the bank can extract higher repayments from the project than can individual investors. But can the bank raise funds to make the loan by borrowing from individual investors?

First consider the project loan as the bank's only asset. In order to determine how much individual investors will lend to the bank, we have to find the liquidation value of the loan in their hands. If a loan sales market does not exist, it will be hard for the bank to transform the loan. So we will first assume zero transformation value for the loan ($b = 0$).²¹ If the bank defaults, individual investors can seize the loan. The "liquidation" value then depends on how much individual investors can extract from the ultimate borrower, the project, given the legal payments due on the loan.

Suppose the intermediary bank defaults at date 1, *after* it has received the first loan payment from the project.²² That payment is cash and the investors cannot reach it. The investors can take the loan, at

different macroeconomic risks, the problem would emerge again unless the exposure is easy to detect.

²¹ In practice, a bank can also transform the quality of its loan portfolio by demanding early repayment and reinvesting the money in poorer quality assets. Or it can cut side deals with its borrowers, asking for early payment in return for debt forgiveness. This implies that the effective transformation possibilities in a loan portfolio increase as the ultimate borrowers obtain more access to funds. For simplicity, we assume the borrower can just meet debt payments at date 1, leaving it with no residual free cash. However, this raises the interesting possibility of differing degrees of transformation risk in the loan portfolio over the business cycle.

²² Another possible contract would require the intermediary to pay investors *before* the payments P_1 and P_2 are received from the project. This contract would give investors more bargaining power. But it would also require the intermediary to have another liquid asset (cash) on hand between dates 0 and 1. This situation is covered by Proposition 2 below.

that point worth $\text{Min} [\alpha^P d_2^P, P_2]$ or they can negotiate with the intermediary for a share of the second payment, obtaining $\alpha^P P_2$. Obviously they will take the loan, and since $P_2 \leq \alpha^P d_2^P$, their payoff will be P_2 .

If the intermediary does not default at date 1, investors get P_1 at date 1 but nothing at date 2. The loan has no value at date 2 -- that is, $d_2=0$ after both payments have been made to the intermediary. In no case can investors extract the full amount of the loan, i.e., $B=P_1+P_2$ from the intermediary. Therefore:

LEMMA 2. If there is no secondary market for the project loan, the bank cannot fully fund the loan on a stand alone basis by borrowing from individual investors.

This lemma shows why intermediation may not dominate direct lending, even when the intermediary has more bargaining power over the project's cash flows. Loans are self-liquidating assets that turn into cash repayments. The individual investor has no power to recover the cash repaid to the intermediary bank if the loan is the intermediary's only asset. The loan, by itself, has little liquidation value in the individual investor's hands, so a bank with the loan as its only asset cannot finance the entire loan from outside investors.²³

Now consider a bank that has core assets other than the loan.

COROLLARY 2. When a bank with core assets and a loan transforms the core assets, individual lenders to the bank are better off seizing the loan than renegotiating the debt.

PROOF OF COROLLARY 2. If the assets are transformed, outside investors cannot reach them. They can only take the loan, worth P_2^P . If they wait until date 2, they will receive nothing. Obviously they will take the loan.

The corollary says that the agency costs of allowing the intermediary to manage the loan, as soon as it becomes stand-alone, are so high that investors are better off eliminating the intermediary and managing the loan themselves, even though they have less bargaining power with the ultimate borrower. The implication is that if the intermediary has both liquid assets and loans, it cannot transform only the liquid assets and hope to renegotiate the debt on its loans.

²³ The extension to a portfolio of loans with more complicated timing is straightforward.

Lemma 2 and corollary 2 imply:

PROPOSITION 2. If there is no secondary market for a loan to an illiquid project, then

(i) Regardless of the bank's bargaining power over the project's cash flows, a_F^P , the incremental debt capacity an illiquid or liquid bank adds with a loan to a project is always less than the amount the project can raise by borrowing directly from individual investors.

(ii) The debt capacity the overly liquid bank adds by making a loan to a project may exceed the debt capacity the project has borrowing directly from investors. If so,

(a) If the overly liquid bank cannot commit not to transform assets selectively, it will have higher debt capacity by making a loan to a project than by undertaking the project in-house.

(b) The debt capacity of the overly liquid bank (weakly) increases in its bargaining power over the project's cash flows, a_F^P .

PROOF OF PROPOSITION 2. See Appendix.

The bank's greater bargaining power over the project ensures that the repayments it can extract from the project exceed the repayments individual investors can obtain by lending directly to the project. So it can lend more. Unfortunately, as lemma 2 indicates, individual lenders to the bank cannot extract this money in its entirety from the bank if the loan is the only asset the bank possesses. Individual investors can threaten to liquidate other assets held by the bank. But it is only when these assets are overly liquid that they have liquidation value which is not already supporting debt. Therefore, even though liquid or illiquid firms have the greater bargaining power than individual investors, and can lend more to projects, they cannot become intermediaries because their greater bargaining power does not enhance the amount they can borrow from the individual investors. Hence proposition 2 (i). But for overly liquid banks, the prospect of future cash inflow from loans will bind more of the (overly) liquid core assets, and can increase the bank's ability to borrow proportionately. Hence proposition 2 (ii). So a bank whose core assets are overly liquid has a comparative advantage over liquid and illiquid banks in

intermediating credit because it can borrow more of the amount needed to make loans. Then 2(ii)a follows immediately from corollary 2 while 2(ii)b is straightforward.

To summarize the basic intuition: Intermediation works because two different forces are at work, one on each side of the bank's balance sheet. The bank can extract more from the project manager because it has greater bargaining power than individual investors, while individual investors can threaten liquidation (which is more effective because the loan makes the bank less "overly" liquid) to extract repayment.

Example (continued): Let $a_F^P = .7 > a_I^P = .5$, as in our earlier example. The bank can make a \$1 loan to the project, with enforceable payments $P_1^P = .7$ and $P_2^P = .3$. The project, borrowing directly from individual investors, can raise only .8 (because $a_I^P < a_F^P$). But can the bank raise money against the loan? On a stand-alone basis, the loan can be thought of as an asset producing cash flows equal to the repayments on the loan, and with liquidation value as derived above. Therefore, $C_1^L = P_1^P = .7$, $C_2^L = P_2^P = .3$, $d_1^L = .3$, $d_2^L = 0$, and $a_I^P = .5$. The debt capacity when the loan is the sole asset is only .3. But because the liquidation value of the loan is small compared to the cash flows it produces, the project loan is a great asset for an overly liquid bank (as in the earlier example, with $C_1^F = 1$, $C_2^F = 1$, $d_1^F = 5$, $d_2^F = 4$, $a^F = .5$, $\alpha^F = 1$). Such a bank can borrow \$1 against every loan of \$1 it makes to a project. In fact, given its stock of (overly) liquid core assets, it can loan to five projects before it loses its comparative advantage in making loans. Intermediation pays, because the projects each get \$ 1 in loans from the bank rather than the \$.8 they can raise directly from individuals. The bank can raise this money directly from individual investors because investors can enforce payments by threatening to liquidate assets (and the bank's liquid core assets provide enough value to make this credible). Their low bargaining power is immaterial. It is also easy to check that the bank will not transform its core assets selectively, because the loans will then be seized by creditors with the attendant loss in value. Finally, note that a liquid or illiquid firm can not raise more than \$.3 against the loan.

All the above is predicated on loans having no secondary market. What if there is one, so that the

bank's loans can be sold to other institutional investors? Not only does this open up transformation possibilities for the bank, it also allows individual lenders to seize loans and sell them to other institutions. To see the other extreme, we will assume an intrinsic liquidity of $\alpha=1$ for the loan and flexibility of $b=1$, so that the loan can be sold for $\text{Min} [a_F^P C_2 + \alpha^P d_2^P, P_2^P]$ if seized at date 1 and 0 after date 2.

COROLLARY 3. When a secondary loan market opens, the incremental debt capacity an overly liquid bank obtains by making a loan is zero.

PROOF OF COROLLARY 3. See Appendix.

When a secondary market opens up, loans can be sold for their intrinsic value. There is a dramatic shift in comparative advantage in financing loans. Loans are too liquid, so the transformation risk constraint is not relaxed at all. The overly liquid bank does not get any additional debt capacity by making the liquid loan and will be dis-intermediated.

B. The Origins of Banking

It is interesting to examine the historical origins of banks (defined as deposit taking intermediaries which also make loans) with our model in mind. De Roover [1948], in his classic history of the evolution of money, banking and credit in medieval Bruges, argues that *moneychangers* were the true precursors of the modern bank. The activity of moneychangers was originally confined to trade in bullion and the exchange of coins. But coins in the Middle Ages were not uniform (because coinage was imperfect), and their quality was hard to evaluate.²⁴ It made sense to minimize the amount of transactions done using coins. Moneychangers started taking in demand deposits primarily to create a payments system. They facilitated trade by making transfers with simple accounting adjustments, without going through the actual process of transferring suspect coins.

²⁴ Gresham's Law ensured that the best coins were kept back while the worst coins -- the most worn, clipped or debased ones -- were culled and offered for trade. An intimate knowledge of coins and the state of finances of the local issuing authority (so as to predict the direction of seigniorage) was a great advantage.

Moneychangers maintained reserves of over 30 percent to service their deposits. The liquidity of the core assets necessary to their business should have resulted in substantial transformation risk. Riu's [1979] study of banking in medieval Aragon is especially illuminating on this. Bank regulation at this time was predominantly aimed, not at preventing bank runs (an issue much of the recent literature on banking seems to emphasize), but at preventing fraud. Money changers typically avoided payment of debt by "concealing resources or transferring them to associates in Italy. Once the transfer was made, they would claim that the coffers were empty and that creditors could not be paid." Despite harsh legal measures, including a decree that fraudulent moneychangers would be executed *sens tota merce*, the problem was ever-present.

The deposit and transfer business was deemed so important that city governments stepped in, at times of substantial private fraud, to offer the service at public banks (Lane and Mueller [1985], Riu [1979]). But the municipal banks were not immune to transformation risk, though it was of a different nature. The municipal banks typically maintained 100% reserves, which ensured their safety in normal times. However, the government helped itself to the municipal bank's liquid assets in times of war or fiscal distress. This led to the repeated failures of the public banks, including even the famed Bank of Amsterdam, the best-run bank of its time (Kindleberger [1993, p. 49]). Thus a 100% reserve ratio simply increased transformation possibilities even if the bank operated in the public domain. Riu [1979, p. 152] argues that in spite of the private bank bankruptcies that preceded the creation of the municipal Taula de Canvi of Barcelona, "the bourgeoisie of the city showed a continuing confidence in, and sometimes even preference for, the private banks...."

How could the private moneychangers commit to being honest with depositors' money, so much so that they were preferred to public banks (and so much so that the municipal Taula in Barcelona could not monopolize the deposit business even after strict limitations were placed on private competition)? Apart from staking their own illiquid reputations, moneychangers made loans to the local community, for example, through unsecured overdrafts to depositors. The overdrafts were not self-liquidating loans.

They could stay open on the books for an indefinite period. Thus borrowers could obtain long-term working capital which was hard for the moneylenders to liquidate surreptitiously in times of stress without alerting depositors. The other main outlet for the moneychanger's funds was direct investment in business such as trading.

Our theory says that the reason the municipal bank did not drive out the private banks was precisely because the municipal bank maintained 100% reserves. Instead of making the bank safe, it merely increased the temptation for the municipality to raid the bank. By contrast, the illiquid loans and investments served to bind the considerable liquid assets the moneychanger needed to service deposits.²⁵

Our explanation of the association between demand deposits and illiquid loans contrasts with the existing literature. Diamond and Dybvig [1983] argue that banks exist because individuals, faced with uncertain consumption needs, want to be able to withdraw savings on demand. Projects, however, may require long-term funding. An individual investing directly in a project will have to sell his claim in a securities market at a low price if he wants cash before the project comes to fruition. Thus a bank can be a form of insurance by which the higher returns from long-term investment are shared, *ex ante*, between those who may need to consume early and those who want to consume late. This, however, presupposes an excess supply of high-return, illiquid projects relative to savings that are not subject to liquidity demands. Our model says that banks can make illiquid loans even when this is not true. Also, in Diamond and Dybvig, the bank is a vehicle by which depositors invest to smooth returns. In our model,

²⁵ It is also possible for the owners of illiquid assets to accept liquid cash deposits for safe-keeping. For instance, the innkeepers of Bruges used to perform this function, perhaps offering their illiquid reputation or inns implicitly as collateral. Similarly, Usher [1943, p. 19] reports that "in many instances, individuals and families [in fourteenth and fifteenth century Europe] owned both a trading and a banking enterprise...it is difficult to decide which of the enterprises was the more important." Our theory can also explain why, in Bruges, the moneychangers accepted the risk of not being able to service deposits by making illiquid loans and left more liquid secured loan opportunities to pawnbrokers. De Roover offers regulation and the opprobrium associated with usury as explanations. But regulations are endogenous, and moneychangers often disguised the interest in their loans. We say that, given the overhang of liquidity from the deposit business, moneychangers had a comparative disadvantage in holding the liquid collateral from pawnbroker loans. Conversely, given the liquidity of their loans, pawnbrokers had a comparative disadvantage in maintaining the liquidity needed to service demand deposits, and had a hard time attracting time deposits. Consequently, they were largely self-financed.

returns may be incidental. The bank may offer only payment services or just safety to depositors, and still exist.

Another strand of the literature starts with the notion that firms are subject to moral hazard, so loans need to be monitored. Free-rider problems and the high monitoring costs for individuals result in delegation of monitoring to banks. The liabilities of the bank are then structured to minimize individuals' costs of monitoring the bank.²⁶ Calomiris and Kahn [1991] argue that the first-come, first-served nature of bank deposits will give large depositors an added incentive to monitor, keeping bank management on the straight and narrow at low cost.

Our theory is not inconsistent with this, but we differ in emphasis and causality. It is because banks have to maintain liquid reserves to service demand deposits that they have a comparative advantage in making illiquid loans. An incidental (though not inconsequential) effect of the demandable nature of deposits is to keep bank management honest. By contrast, the primary role of banks in the papers just discussed is to make loans, and the optimal structure of bank liabilities is, only as a consequence, short term.

C. ... and disintermediation

With technological development and improvements in property rights, the nature of transformation risk shifts away from fraud or theft. Management now have to find ways legal ways to appropriate value. But transformation risk is not eliminated, because the investment opportunities now available to bank managers extend far beyond the community loans that were their historical focus. Managers can substitute from liquid assets into illiquid investments, such as complex derivative positions, that only they know how to manage. Such substitution, as we have argued, is equivalent from the lender's perspective to stealing part of the assets. So even though greater public disclosure and better enforcement has reduced fraud, the transformation risk at modern banks may be as high as ever. This can explain what appears to be disintermediation, the movement of more creditworthy firms away from borrowing

²⁶ See Diamond [1984], Calomiris and Kahn [1991] and Flannery [1994].

from financial intermediaries to direct market financing.

The most creditworthy firms now typically use the commercial paper market for short term funding.²⁷ Historically, banks used to do this lending. The growth of the commercial paper market has led some to suggest that banks are in permanent decline, and their role will be taken by markets. But closer examination (see, for example, Boyd and Gertler [1994]) indicates that banks have simply moved away from directly funding high quality clients, and instead, now focus on providing guarantees and back-up lines for commercial paper. These activities may be nearly as profitable as direct funding (Gertler [1995, p.204]). The question “why disintermediation?” should then be rephrased as “why have banks stopped funding high quality clients directly?”

Our theory offers an explanation. Loans to such clients are very liquid; the loans can be syndicated widely at issue, or sold later in the secondary loan sales market. The client also has the deep pockets to repay or refinance the loan at short notice. The liquidity of these loans increases possibilities that they will be transformed, which in turn implies that the rate of interest demanded by investors in the bank does not respond to the marginal high-quality loan made by the bank.

Consider a bank that is A rated and borrows at 12%. Theoretically, it can make a new loan to a AAA customer at 10% without losing money, because the bank’s existing claimholders, seeing the new safe loan, will reduce their estimate of the bank’s credit risk, and demand a lower rate from the bank. In a Modigliani-Miller world, the downward adjustment on the bank’s average cost of funds is just enough to make the new loan worthwhile. But if bank balance sheets are opaque and transformation risk substantial, the bank’s average cost of capital does not fall sufficiently, because investors do not know whether, or for how long, the new loan will be there. Even if the bank makes a loan to General Electric and holds it up for all to see, creditors do not know if the bank will sell it the next day and reinvest the proceeds in bridge loans to a leveraged buyout, or in a risky proprietary trade.

Banks, especially the lower-rated ones, therefore have a cost disadvantage in funding the highest

²⁷See, for example, Calomiris, Himmelberg and Wachtel [1994] and Gorton and Pennachi [1992].

quality clients. The solution then is for high quality firms to finance directly from the market, for example, through commercial paper. The reason why direct financing works is that investors charge a rate appropriate to the risk of the high quality firm, rather than a rate commensurate with the average risk of the lower credit-quality bank. But banking services such as monitoring may still be valuable (to the firm, and to other creditors). The bank commits to continue providing this service by guaranteeing the commercial paper, so that it is called upon to provide costly funds only on a contingent basis.

More generally, if it is not important to transform liquid assets in the course of business, a borrower is better off placing liquid assets in a separate corporate entity, setting its flexibility to zero, and financing the entity directly from investors. This, in a sense, is *securitization*. For instance, money market mutual funds are constrained through legal means and regulatory oversight to investing in a narrow class of assets, and thus can hold liquid assets that banks cannot hold.

V. Concluding Remarks.

In this paper, we explore how a firm's choice of additional assets may depend on the assets used in its core business. Our theory offers a different way of thinking about intermediation and disintermediation. Policymakers, for example, should note that exogenous changes in liquidity -- the development of a secondary market for bank loans, for example, or expansion of trading of derivatives correlated with banks' traditional assets -- may have substantial impacts on the comparative advantage of financial institutions. The increasing movement of banks into illiquid businesses may be a natural response to changes in the liquidity of banks' core businesses. Changes in regulations affecting the intrinsic liquidity of financial institutions -- such as changes in reserve requirements, changes in the cost of access to the discount window, and changes in the franchise value of liquid businesses like servicing demand deposits -- can affect the other businesses that institutions undertake.

Various implications for firm and creditor behavior have been left unexplored. For instance, firms may not want to take on more liquid new assets because this enhances the value of existing creditors' claims. This is analogous to the under-investment problem (Myers [1977]). Collateralizing the

new assets (or project finance) may be a way of preventing the liquidity of the new assets from spilling over and enhancing the power of old creditors (an idea closely related to Stulz and Johnson [1985]).

We have taken asset liquidity as given. No doubt there are reasons why the intrinsic liquidity of assets would be affected, even in a partial equilibrium setting, by the forces and institutions we examine. We have not attempted to analyze this. Yet there are many plausible situations where the asset's liquidity is exogenous. Our analysis, therefore, has foundation, and we are convinced of its empirical relevance. Our greater concern is the lack of a general equilibrium analysis. In principle, the forces we examine should lower the liquidity premium. Whether they have other effects is left to future research.

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Appendix

Proof of Proposition 1. It is possible that some of the three constraints, (a), (b), and (c), that precede proposition 1 may not bind for any $\alpha \in [0,1]$. To make the problem interesting (and coincidentally, to reduce the number of cases examined), we assume that each constraint binds over a non-empty range so that $0 < \alpha_L < \alpha_{LL} < 1$ for both the project and the firm. To demonstrate the generality of these propositions, we prove them with an arbitrary flexibility parameter, b (see section III).

(a) Denote the stand alone debt capacity for the firm by B^F , for the project by B^P , and for the combined entity by B^{F+P} . We know that debt capacity for any entity is given by

$$(17) \quad \text{Min} \{ \text{Max} [aC_2 + \alpha d_2, \alpha d_1], \quad C_1 + C_2 + d_2 - b\alpha d_1 \}$$

Supposing both firm and project are illiquid so that $B^F = a^F C_2^F + \alpha^F d_2^F$ and $B^P = a^P C_2^P + \alpha^P d_2^P$. Then it is clear that debt capacities for the combined firm will be additive. In other words, because neither liquidation value, αd_1 , nor transformation risk, $C_1 + C_2 + d_2 - b\alpha d_1$, bind for the stand alone firm or project, they should not bind for the combined entity. Similarly for the other cases.

$$(b) \text{ In this case } B^F = \text{Max} [a^F C_2^F + \alpha^F d_2^F, \alpha^F d_1^F] = \alpha^F d_1^F, \quad B^P = \text{Max} [a^P C_2^P + \alpha^P d_2^P, \alpha^P d_1^P] = a^P C_2^P + \alpha^P d_2^P$$

$$\text{Therefore, } B^{F+P} = \text{Max} [a^F C_2^F + \alpha^F d_2^F + a^P C_2^P + \alpha^P d_2^P, \alpha^F d_1^F + \alpha^P d_1^P] \leq \alpha^F d_1^F + a^P C_2^P + \alpha^P d_2^P \leq B^F + B^P$$

Note that transformation risk does not bind on a stand alone basis, and consequently does not bind when the project and firm are combined. Another way of seeing this result is that the maximum of the sums is (weakly) less than the sum of the maximums.

(c) Let the range of projects that the overly liquid firm have an absolute advantage in financing in-house be $[\underline{\alpha}^P, \bar{\alpha}^P]$. We start by arguing that the overly liquid firm can obtain strictly more finance for all liquid

projects than they can get on a stand alone basis. By definition, we know that for an overly liquid firm

$$(18) \quad B^F = C_1^F + C_2^F + d_2^F - b^F \alpha^F d_1^F < \alpha^F d_1^F.$$

Also, because $\alpha_L < \alpha_{LL}$, the liquidation value rather than continuation value would determine debt capacity if transformation risk were not binding. So

$$(19) \quad \text{Max} [a^F C_2^F + \alpha^F d_2^F, \alpha^F d_1^F] = \alpha^F d_1^F.$$

For a liquid project,

$$(20) \quad B^P = \text{Max} [a^P C_2^P + \alpha^P d_2^P, \alpha^P d_1^P] = \alpha^P d_1^P < C_1^P + C_2^P + d_2^P - b^P \alpha^P d_1^P.$$

Because continuation value does not determine debt capacity for either project or firm on a stand alone basis, it should not determine debt capacity when the two are combined. This implies that the combined firm and project is either liquid or overly liquid. If liquid, $B^{F+P} = \alpha^F d_1^F + \alpha^P d_1^P > C_1^F + C_2^F + d_2^F - b^F \alpha^F d_1^F + \alpha^P d_1^P \geq B^F + B^P$. If overly liquid, $B^{F+P} = C_1^F + C_2^F + d_2^F - b^F \alpha^F d_1^F + C_1^P + C_2^P + d_2^P - b^P \alpha^P d_1^P > C_1^F + C_2^F + d_2^F - b^F \alpha^F d_1^F + \alpha^P d_1^P \geq B^F + B^P$.

This proves that there exist projects for which the overly liquid firm can raise more finance when the projects are done in-house than on a stand alone basis. Also, because the overly liquid firm has no comparative advantage in funding overly liquid projects, the highest intrinsic liquidity that can be advantageously financed in-house is the liquid project with the highest intrinsic liquidity. Therefore,

$$(21) \quad \bar{\alpha}^P = \alpha_{LL}^P.$$

Now we determine the lowest quality project that can advantageously be funded by the overly liquid firm. The proof is by construction. Note that from what we have just proved, this project has to be a illiquid one, i.e., $0 \leq \underline{\alpha}^P < \alpha_L$. We first find the lowest project intrinsic liquidity such that the combined entity is just overly liquid. Let this be l . It is the smallest α^P that solves

$$(20) \quad \text{Min} \{ \text{Max} [a^F C_2^F + \alpha^F d_2^F + a^P C_2^P + \alpha^P d_2^P, \alpha^F d_1^F + \alpha^P d_1^P], C_1^F + C_2^F + d_2^F - b^F \alpha^F d_1^F + C_1^P + C_2^P + d_2^P - b^P \alpha^P d_1^P \} = C_1^F + C_2^F + d_2^F - b^F \alpha^F d_1^F + C_1^P + C_2^P + d_2^P - b^P \alpha^P d_1^P.$$

Since (20) is satisfied for $\alpha^P = \alpha_{LL}^P$, by continuity either a positive l exists, or (20) is satisfied even when $\alpha^P = 0$. If so, all illiquid projects can be more advantageously financed within the firm. This is because for the illiquid project, $C_1^P + C_2^P + d_2^P - b^P \alpha^P d_1^P \geq a^P C_2^P + \alpha^P d_2^P$, so the right hand side of (20) is greater than $B^F + B^P$. If $0 < l < \alpha_{LL}^P$, then let $\underline{l} < l$ be the lowest project intrinsic liquidity such that the combined entity is just liquid, i.e., $\underline{l} = (a^F C_2^F + \alpha^F d_2^F + a^P C_2^P + \alpha^P d_2^P - \alpha^F d_1^F) / d_1^P$. If this number is less than zero, set $\underline{l} = 0$. We now check if $B^{F+P} > B^F + B^P$ at $\alpha^P = \underline{l}$. If yes, then a reduction in the liquidity of the project changes neither side of the above inequality (since the combined entity and the stand alone project are both illiquid, their debt capacities vary in the same way with project liquidity). Consequently, all projects of lower liquidity can be advantageously financed in-house and $\underline{\alpha}^P = 0$. If no, we look for the minimum α^P such that $\underline{l} < \underline{\alpha}^P < l$ and

$$(23) \quad \alpha^F d_1^F + \alpha^P d_1^P \geq C_1^F + C_2^F + d_2^F - b^F \alpha^F d_1^F + a^P C_2^P + \alpha^P d_2^P.$$

Thus $\underline{\alpha}^P = [C_1^F + C_2^F + d_2^F - b^F \alpha^F d_1^F - \alpha^F d_1^F + a^P C_2^P + \alpha^P d_2^P] / d_1^P$. Since (23) is certainly true at $\alpha^P = l$, and the left hand side of (23) increases at a faster rate with α^P than the right hand side, by continuity, $\underline{\alpha}^P$ must exist.

Q.E.D.

Proof of corollary 1. From the construction in the proof to Proposition 1 (c), we know that $\bar{\alpha}^P = \alpha_{LL}$ which does not depend on the intrinsic liquidity of the firm. Also $\underline{\alpha}^P=0$ or $\underline{\alpha}^P = [C_1^F + C_2^F + d_2^F - b^F \alpha^F d_1^F - \alpha^F d_1^F + a^P C_2^P + \alpha^P d_2^P]/d_1^P$. So $\underline{\alpha}^P$ (weakly) decreases in α^F . We have not yet determined α^{P*} . We determine it by construction. First we make the following assertions which follow directly from definitions:

1. If for a given α^P , the combination of the (liquid or illiquid) project and the firm is overly liquid, then the difference between combined debt capacity and the sum of stand-alone debt capacities can be increased by decreasing α^P (unless $\alpha^P = 0$). This is because a marginal decrease in intrinsic liquidity of the project will (weakly) decrease its stand-alone debt capacity and increase combined debt capacity. If the combined entity is overly liquid even when $\alpha^P=0$, then $\alpha^{P*} = 0$. Otherwise, α^{P*} cannot be such that the combined entity is overly liquid.
2. Over the range of project liquidity where both the project and the combined entity are liquid, there is no change in the difference between combined debt capacity and the sum of stand-alone debt capacities with changes in project liquidity. This is because an increase in the liquidity of the project increases both combined debt capacity and stand-alone debt capacity by the same amount.
3. If the combined entity is liquid when the project is illiquid, the difference between combined debt capacity and the sum of stand-alone debt capacities increases in the intrinsic liquidity of the project. This is because an increase in the liquidity of the project increases combined debt capacity by more than it increases stand-alone debt capacity.

4. The combined entity cannot be illiquid when the project is liquid (because the firm is overly liquid). So the final possibility is that the combined entity is illiquid when the project is illiquid, in which case there is no change in the difference between combined debt capacity and the sum of stand-alone debt capacities with a change in project liquidity.

From 1, 2, 3, and 4, the effect of changes in project liquidity on the difference between combined debt capacity and the sum of stand-alone debt capacities depend on whether the project is liquid or illiquid, and on whether the combined entity is overly liquid, liquid or illiquid. The project liquidity that results in the largest difference is easily found by considering the combined debt capacity when $\alpha^P = \alpha_L$. Recall that this is the level of intrinsic project liquidity that separates liquid projects from illiquid projects.

Case 1: If B^{F+P} is overly liquid when $\alpha^P = \alpha_L$, the difference between combined debt capacity and stand alone capacity can be increased by reducing α^P (assertion 1) upto the point that the combined entity is just overly liquid. As defined in the proof to proposition 1, this is $\alpha^P = l$. Note that the project is illiquid when α^P is below α_L , so the difference cannot be enhanced by decreasing project liquidity further (assertion 3 and 4). Therefore α^{P*} is the highest intrinsic liquidity that equates $\text{Max} [a^F C_2^F + \alpha^F d_2^F + a^P C_2^P + \alpha^P d_2^P, \alpha^F d_1^F + \alpha^P d_1^P]$ to $C_1^F + C_2^F + d_2^F - b^F \alpha^F d_1^F + C_1^P + C_2^P + d_2^P - b^P \alpha^P d_1^P$ or 0, whichever is higher.

Case 2: If B^{F+P} is liquid when $\alpha^P = \alpha_L$, from assertion (2) and assertion (3), there is a range of project liquidity $\alpha^{P*} \in [\alpha_L, l]$ where the difference between combined debt capacity and stand alone capacity is maximized. l which is defined above is the maximum project liquidity such that the combined entity is liquid.

Finally, the combined entity cannot be illiquid, given that the firm is overly liquid, and the project is on the borderline. Thus α^{P*} is either a unique value or a range, and it is easily checked that if unique, it is weakly decreasing in α^F , and if a range, the upper limit is decreasing in α^F .

Q.E.D.

Proof of Proposition 2. (i) By borrowing directly from investors, the project can raise $a^P C_2 + \alpha^P d_2^P$. When a firm with liquid or illiquid assets borrows against a loan to the project, it will be able to raise at most the higher of the liquidation value or going concern value of the loan. But from the reasoning leading to lemma 2, this is P_2^P which is (weakly) less than $\alpha^P d_2^P$. So a liquid or illiquid firm will be able to borrow less against the loan than the project can borrow directly.

(ii) We only have to show that it is possible for an overly liquid firm to add more debt capacity making a loan to a project than the project's stand alone debt capacity. Let the assets of the overly liquid firm be such that

$$(24) \quad C_1^F + C_2^F + d_2^F - 2\alpha^F d_1^P + (a^P C_2^P + \alpha^P d_2^P) < 0$$

This implies that the core assets of the overly liquid firm are large compared to the cash flows of the firm or the project. The project loan increases the cash flows to the firm by $P_1^P + P_2^P = a^P C_2^P + \alpha^P d_2^P$, and loosens the transformation constraint by exactly this amount. But (24) ensures that debt capacity is still determined by transformation risk, even after the loan to the project is added to assets. So the incremental debt capacity the loan adds to the firm is $P_1^P + P_2^P = a^P C_2^P + \alpha^P d_2^P > a^P C_2 + \alpha^P d_2^P$ where the last term in the inequality is the debt capacity of the project on a stand alone basis.

(ii a) Follows directly from corollary 2.

(ii b) Follows because $P_1^{*P} + P_2^{*P}$ increases in a^P .

Q.E.D.

Proof of Corollary 3. When the loan can be sold for its full face, the transformation value of the loan is exactly equal to the repayments that are due on it. So the loan does not reduce the transformation constraint at all. Hence the debt capacity contributed by the loan to a firm with overly liquid core assets is zero.

Q.E.D.